THE SIGNIFICANCE OF ANNUAL TEMPERATURE DIFFERENTIALS IN THE DESIGN OF STRUCTURES FOR CLIMATIC EFFECTS

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Inzhenerno-Fizicheskii Zhurnal, Vol. 8, No. 2, pp. 190-197, 1965

A method is suggested for determining the mean annual temperature differential for a single-layer slab in an unheated building in the design of structures for climatic temperature effects.

Variation of the temperature of the outside air and the intensity of solar radiation is accompanied by a change in the mean temperature of the structural elements of unheated buildings, which are least favorable from the point of view of extreme annual temperature differentials.

The CNS (Soviet Construction Norms and Specifications) give no instructions regarding the choice of temperature differentials in designing buildings for climatic temperature stresses, but merely specify the distances between expansion joints.

Available climatic data permit the determination of the boundary conditions for calculating temperature fields in structural elements, and hence the determination of temperature differentials for designing structures for climatic temperature stresses.

However, the boundary conditions (temperature of outside air and intensity of solar radiation) are only statistical means, so the accuracy of the solutions is limited.

An important special case is the determination of the annual temperature differential for slab-type elements. In construction this is important for determining the climatic loads on the supporting columns of industrial buildings due to thermal deformation of the roof, for determining the climatic loads on walls, and in a number of other cases.

As indicated above, climatic temperature stresses are the result of the action on the slab of the annual fluctuations in air temperature and the total flux of solar radiation (direct and indirect), which is taken into account only for the summer months.

Variations in the mean slab temperature cause uniform strains in the plane of the slab; the stresses produced in the slab depend on how it is supported.

It will be assumed that the thermal waves from the annual air temperature fluctuations are not damped in the slab (for thick slabs this assumption is not fully valid, but the damping of the annual waves can be calculated by the method suggested below), and therefore that the mean temperature of the slab is equal to the mean temperature of the air during the month in question.

It will also be assumed that in the course of 24 hours the air temperature varies sinusoidally, with an amplitude equal to the mean amplitude of the diurnal temperature fluctuations during the month in question.

The latter assumption is justified by the following considerations. In the design stage it is difficult to foresee possible combinations of the state of erection of the structure and variations of the outside air temperature. Thus the mean temperature of the elements of a structure at the time when it becomes statically indeterminate is unknown. If a building is enclosed, for example, during the winter, this may coincide either with severe frosts or with thaws. Therefore, only the most stable climatic factors should be introduced into the calculations. As regards the diurnal fluctuations of the air temperature, the appropriate factor is the mean 24-hour amplitude of the outside air temperature given in CNS II-A, 6-62, Table 2. The maximum monthly 24-hour amplitudes of the air temperature will also be found there. These are observed only once a month, however, and are therefore not suitable for design purposes.

The proposed method has been brought to the stage of numerical solution for a single-layer slab. The reason for this is that the majority of buildings under construction, including those with large prefabricated panels, have virtually single-layer walls (not counting the texture layer); the roof slabs of industrial buildings may also be considered as single-layer, since in the majority of cases the insulation is added a considerable time after the erection of the slab; the building may thus remain for six months or more unheated and exposed to the full annual differential of climatic thermal loading under the most unfavorable conditions.

Under the assumptions made above, the solution of the problem of determining the amplitude of the mean temperature of the slab $\vartheta_m = \frac{1}{\delta}\int_0^\delta \vartheta \, dx$ reduces to the solution of three partial problems: a) determination of the mean slab temperature due to a constant heat flux at its outside surface for zero air temperature and convective heat transfer with different coefficients α_o and α_i; b) determination of the amplitude of the mean slab temperature for a sinusoidal heat flux at the same surface, zero air temperature and convective heat transfer with coefficients α_o and α_i; c) determination of the amplitude of the mean slab temperature for harmonic oscillations of the air temperature at both faces of the slab with the same coefficients α_o and α_i. The system of equations for the solution of the first problem has the form

$$\frac{\partial^2 \vartheta}{\partial x^2} = 0, \quad 0 \leqslant x \leqslant \delta;$$

$$\lambda \frac{\partial \vartheta}{\partial x} + q_{st}\rho - \alpha_o \vartheta = 0 \quad \text{when } x = 0,$$

$$\lambda \frac{\partial \vartheta}{\partial x} + \alpha_i \vartheta = 0 \quad \text{when } x = \delta.$$

The mean temperature of the slab is then

$$\vartheta_m = q_{st}\rho \left(\lambda + \frac{\alpha_i \delta}{2}\right) \quad [\lambda(\alpha_o + \alpha_i) + \alpha_o \alpha_i \delta]^{-1}. \tag{1}$$

It should be noted that as the slab thickness decreases, i.e., when $\delta \to 0$, $\vartheta_m^{am} \to q_{st}\rho/(\alpha_o + \alpha_i)$. The system of equations for solution of the second problem has the form

$$\frac{\partial \vartheta}{\partial \tau} = a \frac{\partial^2 \vartheta}{\partial x^2}, \quad 0 \leqslant x \leqslant \delta;$$

$$\lambda \frac{\partial \vartheta}{\partial x} + q_0 \rho \sin k\tau - \alpha_o \vartheta = 0 \quad \text{when } x = 0,$$

$$\lambda \frac{\partial \vartheta}{\partial x} + \alpha_i \vartheta = 0 \quad \text{when } x = \delta.$$

We shall seek a solution of the first of these equations in the form

$$\vartheta = A \sin k\tau + B \cos k\tau. \tag{2}$$

We then find that $A = C_1h_1 + C_2h_2 + C_3h_3 + C_4h_4$, $B = C_4h_1 + C_3h_2 - C_2h_2 - C_1h_1$, where $h_1 = \sin px \,\mathrm{sh}\, px$, $h_2 = \sin px \,\mathrm{ch}\, px$, $h_3 = \cos px \,\mathrm{sh}\, px$, $h_4 = \cos px \,\mathrm{ch}\, px$; $\sqrt{\pi/aT}$; $a = \lambda/c\gamma$.

We shall further assume that, when $x = \delta$, $h_1, \ldots, h_4$ are equal to H_1, H_2, H_3, H_4, respectively. The arbitrary constants C_1, C_2, C_3, C_4 are determined from the two remaining equations of the system in question. If we write

$$D = p(H_3 - H_2) + \frac{\alpha_o + \alpha_i}{\lambda} H_4 + \frac{\alpha_o \alpha_i}{2p\lambda^2}(H_2 + H_3),$$

$$E = p(H_3 + H_2) + \frac{\alpha_o + \alpha_i}{\lambda} H_1 + \frac{\alpha_o \alpha_i}{2p\lambda^2}(H_2 - H_3),$$

$$G = q_0 \rho \left[\frac{\alpha_i}{2p\lambda^2}(H_2 + H_3) + \frac{1}{\lambda} H_4\right],$$

$$L = q_0 \rho \left[\frac{\alpha_i}{2p\lambda^2}(H_2 - H_3) + \frac{1}{\lambda} H_1\right],$$

then the arbitrary constants are expressed by the equations:

$$C_1 = \frac{GE - DL}{E^2 + D^2}; \quad C_2 = \frac{\alpha_o}{2p\lambda}(C_4 + C_1) - \frac{q_0\rho}{2p\lambda};$$

$$C_3 = \frac{\alpha_o}{2p\lambda}(C_4 - C_1) - \frac{q_0\rho}{2p\lambda}; \quad C_4 = \frac{EL + GD}{E^2 + D^2}.$$

The amplitude of the oscillations of mean slab temperature is

$$\vartheta_m^{am} = \frac{1}{2p\delta}\{[C_1(H_2-H_3)+C_2(H_1-H_4+1)+C_3(H_1+H_4-1)+ + C_4(H_2+H_3)]^2+[C_4(H_2-H_3)+C_3(H_1-H_4+1)- - C_2(H_1+H_4-1)-C_1(H_2+H_3)]^2\}^{1/2}. \quad (3)$$

Let us analyze the solution for $p\delta \to 0$. In this case the temperature field will be stationary at any moment of time, and the amplitude of the oscillations of the mean slab temperature must tend to $q_0\rho(\alpha_o + \alpha_i)^{-1}$.

Thus, when $p\delta \to 0$ $D \to (\alpha_o + \alpha_i)/\lambda$, $E \to 0$, $G \to q_0\rho/\lambda$, $L \to 0$, $C_1 \to 0$, $C_4 \to GD/D^2 = G/D = q_0\rho/(\alpha_o + \alpha_i)$ and in Eq. (2), when $p\delta \to 0$ $A \to C_4 \times q_0\rho/(\alpha_o + \alpha_i)$, while $B \to (-C_1) \to 0$. Thus, when $p\delta \to 0$ $\vartheta_m = [q_0\rho/(\alpha_o + \alpha_i)] \times \sin k\tau$. This can be shown in the same way for Eq. (3), if when $p\delta \to 0$ the functions H_1, H_2, H_3, H_4 are expanded in powers of $p\delta$. Thus, for a slab with low thermal inertia:

$$\vartheta_m \simeq q_0\rho/(\alpha_o + \alpha_i). \quad (4)$$

The system of equations for the solution of the third problem has the form

$$\frac{\partial \vartheta}{\partial \tau} = a\frac{\partial^2 \vartheta}{\partial x^2}, \quad 0 \leqslant x \leqslant \delta;$$

$$\lambda\frac{\partial \vartheta}{\partial x} - \alpha_o[\vartheta - \vartheta_0 \sin k\tau] = 0 \quad \text{when } x = 0;$$

$$\lambda\frac{\partial \vartheta}{\partial x} + \alpha_i[\vartheta - \vartheta_0 \sin k\tau] = 0 \quad \text{when } x = \delta.$$

The result of solving this problem is similar to the foregoing solution; the coefficients D and E have the same form, $q_0\rho$ is replaced by $\alpha_o\vartheta_0$ in the coefficients G_o' and L', and the term $\alpha_i\vartheta_0/\lambda$ is added in the coefficient G'. Then

$$C_1 = \frac{G'E - L'D}{E^2 + D^2}; \quad C_4 = \frac{L'E + G'D}{E^2 + D^2};$$

$$C_3 = \frac{\alpha_o}{2p\lambda}(C_4 - C_1) - \frac{\alpha_o\vartheta_0}{2p\lambda}; \quad C_2 = \frac{\alpha_o}{2p\lambda}(C_4 + C_1) - \frac{\alpha_o\vartheta_0}{2p\lambda}.$$

The expression for the amplitude of the mean slab temperature (3) does not change. Let us analyze the solution when $p\delta \to 0$. It is clear from physical considerations that in this case the amplitude of the mean slab temperature must tend to ϑ_0.

Thus, when $p\delta \to 0$ $D \to (\alpha_o + \alpha_i)/\lambda$, $E \to 0$, $G' \to (\alpha_o + \alpha_i)/(\lambda)\vartheta_0$, $L \to 0$, $C_1 \to 0$, $C_4 \to GD/D^2 = G/D = \vartheta_0$, and in (2), when $p\delta \to 0$, $A \to C_4 \to \vartheta_0$, while $B \to (-C_1) \to 0$. In this case

$$\vartheta_m^{am} \simeq \vartheta_0. \quad (5)$$

Thus, for a slab with low thermal inertia, the temperature differential may be evaluated from the formula

$$\Delta t = t_{VI} - t_I + \frac{1}{2}(\vartheta_I + \vartheta_{VI}) + \frac{q_\Sigma \rho}{\alpha_o + \alpha_i}. \quad (6)^*$$

Values of t_I and t_{VI}, ϑ_I and ϑ_{VI}, q_Σ, α_o and α_i are given in CNS II-A, 6-62 and 7-62.

In the equation for Δt the amplitudes of the mean slab temperature due to the effect of solar radiation and fluctuations of the air temperature are added, i.e., it is assumed that the fluctuations are in phase. In fact, they are displaced by approximately 3 hours, and the maximum of the air temperature lags behind that of the solar radiation. It is a simple matter to allow for this, but its influence on the final result is insignificant.

The prestressed concrete roof slabs of industrial buildings are 25 cm thick. Let us determine the annual temperature differential for the Moscow area. According to CNS, $t_I = -10.3°$, $t_{VI} = 15.4°$, $\vartheta_I = 7.8°$, $\vartheta_{VI} = 13.3°$, $q_\Sigma = 714 \cdot 1.163$ w/m², for concrete $\rho = 0.65$, $\alpha_o = 20 \cdot 1.163$ w/m² · degree, $\alpha_i = 7.5 \cdot 1.163$ w/m² · degree. We obtain $\Delta t = 53.2$ degrees.

*In CNS II-A, 6-62 double amplitudes of the diurnal fluctuations of the outside air temperature are given from the standpoint of the calculations performed.

A temperature differential of this kind, when the distances between expansion joints are great (of the order of 100 m), can impose severe loads on the columns, which must be taken into account in their design.

Thus (6) may be used as a first approximation for elements with low thermal inertia. Physically, this means that there is a stationary temperature distribution in the slab at any moment of time, and therefore the thickness, and the properties of the material of the slab, do not have an influence on the amplitude of the oscillations of mean slab temperature; the latter is determined only by the intensity of the outside temperature influences and the heat transfer conditions.

It is more complicated to determine the climatic heat loads, when the thermal inertia of the slab must be taken into account. To simplify the discussion and calculations, it is useful to introduce generalized coordinates, in this case the similarity criteria of the thermal processes.

It can be established from dimensional analysis of the solutions obtained that the amplitude of the mean slab temperature in the case of an harmonic heat flux enters into the Ki number and depends on Bi_o, Bi_i, and $p\delta$:

$$\frac{\vartheta_m^{am}\lambda}{q_0\rho\delta} = f\left[\frac{\alpha_o\delta}{\lambda};\ \frac{\alpha_i\delta}{\lambda};\ \delta\sqrt{\frac{\pi}{aT}}\right]. \tag{7}$$

In the case of harmonic oscillations of the air temperature, the amplitude of the mean slab temperature, as a fraction of the amplitude of the air temperature oscillations, depends on these same parameters:

$$\vartheta_m^{am}/\vartheta_0 = \varphi\,[Bi_o,\ Bi_i,\ p\delta]. \tag{8}$$

In determining the form of relations (7) and (8), it was assumed that $\alpha_o = 20 \cdot 1.163$, $\alpha_i = 7.5 \cdot 1.163$ w/m^2 · degree, and Bi varies only with δ and λ; since the slab is single-layer, it is sufficient to assign just one of the two parameters, for example Bi_o.

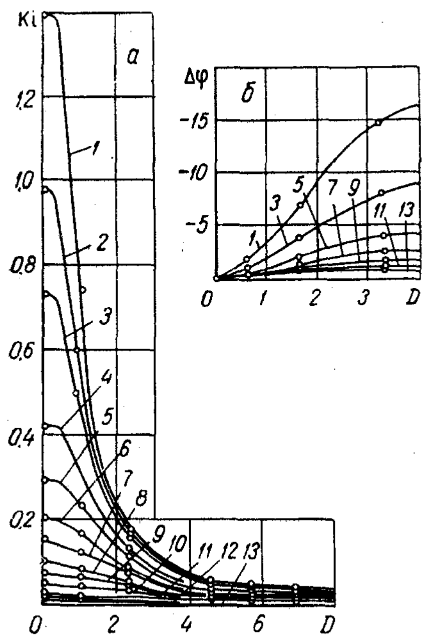

Fig. 1. Ki number (a) and phase shift $\Delta\varphi$ of mean slab temperature relative to phase of heat flux of solar radiation (b) as a function of thermal inertia of slab D: 1 – $Bi_1 = 0.5$; 2 – 0.75; 3 – 1.0; 4 – 1.75; 5 – 2.5; 6 – 3.75; 7 – 5.0; 8 – 7.5; 9 – 10; 10 – 15; 11 – 20; 12 – 30; 13 – 40.

On the basis of the calculations, graphs expressing relations (7) and (8) were constructed (Figs. 1a and 2a). The phase shift of the oscillations of the mean slab temperature relative to the phase of the heat flux and the phase of the air temperature oscillations are given in Figs. 1b and 2b.

The behavior of the solar radiation in June may be described to a first approximation by the equations:

$$q = q_0 \sin\frac{4\pi}{3T}\tau \text{ when } 0 \leqslant \tau \leqslant \frac{3}{4}T = 18 \text{ hr};$$

$$q = 0 \quad \text{when} \quad \frac{3}{4}T \leqslant \tau \leqslant T = 24 \text{ hr}.$$

To obtain this flux it is sufficient to take three terms of the Fourier series:

$$q = \frac{3}{2\pi}q_0 + \frac{6}{5\pi}q_0 \sin\frac{2\pi}{T}\tau - \frac{6}{5\pi}\cos\frac{2\pi}{T}\tau =$$

$$= \frac{3}{2\pi}q_0 + \frac{6}{5\pi}q_0 \sin\frac{2\pi}{T}\tau - \frac{6}{5\pi}q_0 \sin\left(\frac{2\pi}{T}\tau + \frac{\pi}{2}\right). \tag{9}$$

Let us determine, for example, the annual temperature differential for the Moscow area using the following data: $\delta = 20$ cm, $\lambda = 0.6 \cdot 1.163$ w/m · degree, $c = 0.84 \cdot 10^3 \gamma$/kg · degree, $\gamma = 600$ kg/m^3, $\lambda/c\gamma = 5 \cdot 10^{-3}$ m^{2}hr, $\rho = 0.65$.

From (1), taking into account the coefficient $3/2\pi$, we have $\vartheta_m = 6.4$ degree. The quantity $p\delta = 1.05$; $Bi_o = 6.7$. For these values, we find from Fig. 1a, $\vartheta_m^{am}\lambda/q_0\rho\delta = 0.084$, or, taking into account the coefficient $6/5\pi$ of the expansion, we have $\vartheta_m^{am} = 5$ degrees. From Fig. 1b the phase shift $\varphi \simeq 0.5$ degree.

The third term of the expansion gives the same value $\vartheta_m^{am} = 5$ degrees, but with a phase lead of $\pi/2$. The difference is $\vartheta_m^{(2)} = 7.1$ degrees with a phase shift of 45.5°.

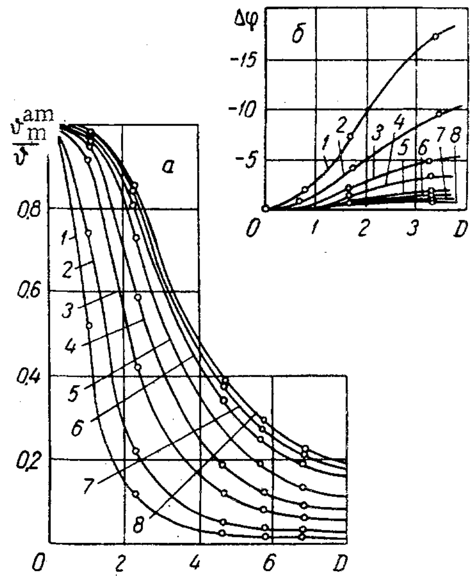

Fig. 2. Ratio of amplitude of mean slab temperature to amplitude of air temperature (a) and phase shift of mean slab temperature relative to phase of air temperature oscillations (b) as a function of thermal inertia of slab: 1 — $Bi_1 = 0.5$; 2 — 1.0; 3 — 2.5; 4 — 5; 5 — 10; 6 — 20; 7 — 30; 8 — 40.

From Fig. 2a we find $\vartheta_m^{am}/\vartheta_0 = 0.89$, and a phase shift of approximately 0.6 degree (Fig. 2b). The half-sum of the mean January and June amplitudes of the air temperature oscillations for Moscow is equal to 10.55 degrees. Thus, from the diurnal oscillations of the air temperature $\vartheta_m^{(3)} = 0.89 \cdot 10.55 = 9.4$ degrees.

A check on the damping of the annual variation of the outside air temperature in the slab may be obtained by calculating the value of $p\delta$ with $T = 24 \cdot 365$ hr, which gives $(p\delta)_{ann} = 0.055$.

It can be seen from the graph that, for the given $p\delta$ and $Bi = 6.7$, $\vartheta_m^{am}/\vartheta_0 = 1$, i.e., the annual temperature wave in the slab is not damped; therefore the monthly mean temperature for January and June will be taken without correction factors.

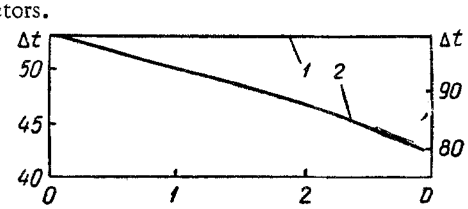

Fig. 3. Comparison of results of calculations of annual temperature differentials for the Moscow area: 1 — approximate method, 2 — exact method.

The air temperature maximum lags by approximately three hours the solar radiation maximum; we shall therefore add to the phase shift from the air temperature oscillations a further $-2\pi(3/24) = -\pi/4$, i.e., the phases of the oscillations of the mean slab temperature due to solar radiation and air temperature practically coincide. Finally, we have: $\Delta t = t_{VI} - t_I = \vartheta_m^{(1)} + \vartheta_m^{(2)} + \vartheta_m^{(3)} = 48.6$ degrees. The result obtained is less than that calculated from (6) by altogether only 9.5%. Figure 3 shows the discrepancy between the annual temperature differentials for the Moscow area calculated by the approximate and exact methods. As the thermal inertia of the slab increases, the exact method naturally gives smaller values of the calculated annual temperature differential, but we may use the simpler approximate formula (6) when $D \leq 2$, for walls classified as "light" in the CNS. The thermal inertia D is linked with $p\delta$ by the relation $D = \sqrt{2}\, p\delta$.

NOTATION

T — period of harmonic oscillations; ϑ — temperature; ϑ_0 — amplitude of diurnal oscillations of air temperature; q_{st} — steady heat flux; q_0 — amplitude of harmonic oscillations of heat flux; q_Σ — maximum hourly value of total flux of direct and indirect solar radiation; ρ — absorption coefficient for solar radiation in slab material; ϑ_m — mean slab temperature; ϑ_m^{am} — amplitude of oscillations of mean slab temperature; α_o and α_i — heat transfer coefficients at outside and inside faces; λ, c, γ — thermal conductivity, specific heat, and specific weight of slab material; δ — slab thickness; t_{VI} and t_I — monthly mean air temperatures for June and January; ϑ_{VI} and ϑ_I — mean amplitudes of diurnal oscillations of air temperature in June and January; Δt — theoretical annual temperature differential.

13 May 1964

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